ECON 577 Homework 5

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Problem: Hull 4.3. The 6-month and 1-year zero rates are both 5% per annum. For a bond that has a life of 18 months and pays a coupon of 4% per annum (with semiannual payments and one having just been made), the yield is 5.2% per annum. What is the bond's price? What is the 18-month zero rate? All rates are quoted with semiannual compounding.

Sol. We see that if a payment has just been made, the bond will issue coupons of $2 at times $t = 0, 6, 12, 18$ (months) and at $t = 18$ month the bond will have its face value of 100 available. Therefore considering discrete discounting,

$$p = 2 + \frac{2}{1 + \frac{0.104}{2}} + \frac{2}{(1 + \frac{0.104}{2})^2} + \frac{102}{(1 + \frac{1.104}{2})^3} = 93.32.$$

Regarding the zero rate, we wish to consider the equation

$$2 + \frac{2}{(1 + \frac{0.10}{2})} + \frac{2}{(1 + \frac{0.10}{2})^2} + \frac{102}{(1 + \frac{x}{2})^3} = p.$$

This returns $(1 + \frac{x}{2})^3 = 1.16439 \Rightarrow x \approx 0.104$, or 10.4%. This matches our expectation as we expect the 18-month rate, where the discount of the face value happens, to align generally with the overall yield rate.

Problem: Hull 4.5.

Maturity (months)	Rate (% per annum)
3	3.0
6	3.2
9	3.4
12	3.5
15	3.6
18	3.7

Calculate forward interest rates for the second, third, fourth, fifth, and sixth quarters.

Sol. We use the formula

$$R_f = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

to calculate the forward rate. Taking t in increments of $1/4$ (for 3 months), we get the forward interest rates as $R_{f,2} = 3.4$, $R_{f,3} = 3.8$, $R_{f,4} = 3.8$, $R_{f,5} = 4.0$, $R_{f,6} = 4.2$, in percentage.

Problem: Hull 4.6. Assuming that risk-free zero rates are as in problem 4.5, what is the value of an FRA where the holder will pay LIBOR and receive 4.5% quarterly compounded for a three-month period starting in one year on a principal of \$1000000? The forward LIBOR rate for the three-month period is 5% quarterly compounded.

Sol. We use the formula

$$V_{FRA} = L(R_k - R_f)(T_2 - T_1)e^{-R_2 T_2}$$

to calculate the value of the FRA. Specifically, with principal $L = 1M$ and spot rate at the 15-th month (corresponding to three months after one year) at 3.6%, we have that

$$V_{FRA} = 1M \times (0.045 - 0.05)\frac{1}{4}e^{-0.036\times 1.25} \approx -1195.00.$$

Problem: Hull 4.11. Suppose a 6-month, 12-month, 18-month, 24-month, and 30-month zero rates are respectively 4%, 4.2%, 4.4%, 4.6%, and 4.8% per annum, with continuous compounding. Estimate the cash price of a bond with a face value of 100 that will mature in 30 months and pay a coupon of 4% per annum semiannually.

Sol. We use the continuous discounting formula:

$$p = 2e^{-0.04\times 0.5} + 2e^{-0.042\times 1} + \cdots + 102e^{-0.048\times 2.5} \approx 98.04.$$

Problem: Hull 4.12. A 3-year bond provides a coupon of 8% semiannually and has a cash price of 104. What is the bond's yield?

Sol. We suspect the discount factor is in the range of $[1, 1.1]$ per annum. Therefore we essentially solves the following equation in MATLAB, see the following code:

```
f = @(x) 4*exp(-0.5*x)+4*exp(-1*x)+4*exp(-1.5*x)+4*exp(-2*x)+4*exp(-2.5*x)+104*exp(-3*x)-104;
x1 = 0;
x2 = 0.1;
epsi = 5 * 10^(-6);
delta = 5 * 10^(-6);
maxf = 100;
sol = secant(f,x1,x2,epsi,delta,maxf);
disp(sol);
disp(f(sol(1)));
```

Here `secant` is the function for secant method, a method commonly used for root estimation. This gives $x = 0.0641$, i.e., the yield is 6.41%.

Problem: Hull 4.22. A 5-year bond with a yield of 7% (continuously compounded) pays an 8% coupon at the end of each year.

(a) What is the bond's price?

(b) What is the bond's duration?

(c) Use the duration to calculate the effect on the bond's price of a 0.2% decrease in its yield.

(d) Recalculate the bond's price on the basis of a 6.8% per annum yield and verify that the result is in agreement with your answer to (c).

Sol. Bond price is given by

$$p = 8e^{-0.07} + 8e^{-0.14} + 8e^{-0.21} + 8e^{-0.28} + 108e^{-0.35} \approx 103.05.$$

The duration of the bond can then be calculated using `sumproduct`, or manually (lol):

$$D = \frac{8e^{-0.07} + 2 \times 8e^{-0.14} + 3 \times 8e^{-0.21} + 4 \times 8e^{-0.28} + 5 \times 108e^{-0.35}}{103.05} \approx 4.32.$$

If the yield decreases by 0.2%, we see from the formula that

$$\frac{\Delta B}{B} = -D \cdot \Delta y = -4.32 \times -0.20\% = +0.86\%$$

to see the bond price will increase by approximately 0.86%. That is, the new bond price will be

$$p' = 103.05 \times 1.0086 \approx 103.94.$$

Problem: Hull 5.3. Suppose that you enter into a 6-month forward contract on a non-dividend-paying stock when the stock price is $30 and the risk-free interest rate (with continuous compounding) is 5% per annum. What is the forward price?

Sol. The forward price is simply $p = 30e^{0.05 \times 0.5} \approx 30.76$.

Problem: Hull 5.4. A stock index currently stands at 350. The risk-free interest rate is 4% per annum (with continuous compounding) and the dividend yield on the index is 3% per annum. What should the futures price for a 4-month contract be?

Sol. The future price is $F = S_0 e^{(r-q)T} = 350e^{(0.04-0.03)\times\frac{1}{3}} \approx 351.17$.

Problem: Hull 5.9. A 1-year long forward contract on a non-dividend-paying stock is entered into when the stock price is \$40 and the risk-free rate of interest is 5% per annum with continuous compounding.

(a) What are the forward price and the initial value of the forward contract?

(b) Six months later, the price of the stock is \$45 and the risk-free interest rate is still 5%. What are the forward price and the value of the forward contract?

Sol. The forward price initially is $p = 40e^{0.05} \approx 42.05$.
Six months later, the forward price is $p' = 45e^{0.05\times 0.5} \approx 46.14$. Therefore the value of the forward contract is

$$V = (46.14 - 42.05) \times e^{-0.5\times 0.05} \approx 3.99.$$

Problem: Hull 5.10. The risk-free rate of interest is 7% per annum with continuous compounding, and the dividend yield on a stock index is 3.2% per annum. The current value of the index is 150. What is the 6-month futures price?

Sol. Again the futures price can be obtained by

$$p = 150e^{(0.07-0.032)\times 0.5} \approx 152.88.$$

Problem: Hull 5.12. Suppose that the risk-free interest rate is 6% per annum with continuous compounding and that the dividend yield on a stock index is 4% per annum. The index is standing at 400, and the futures price for a contract deliverable in four months is 405. What arbitrage opportunities does this create?

Sol. The four months futures price is $400e^{(0.06-0.04)\times\frac{1}{3}} \approx 402.68 < 405$. There exist arbitrage opportunities as the derivative is overpriced. One could short futures and long index to arbitrage.

Problem: hull 5.15. The spot price of silver is \$25 per ounce. The storage costs are \$0.24 per ounce per year payable quarterly in advance. Assuming that interest rates are 5% per annum for all maturities, calculate the futures price of silver for delivery in 9 months.

Sol. We first calculate the present value of the storage cost. Specifically,

$$C = 0.06 + 0.06e^{-0.025\times 0.05} + 0.06e^{-0.5\times 0.05} \approx 0.18.$$

Therefore the futures price will have to add the storage cost, which gives

$$F_0 = (25 + 0.18)e^{0.05\times\frac{3}{4}} \approx 26.14.$$